Auction 107 Assignment Phase Technical Guide

1 Introduction

This technical guide sets forth the details of the bidding procedures for the second phase—the assignment phase—of Auction 107 as described in the *Auction 107 Procedures Public Notice*.[1] The assignment phase offers winners of generic spectrum blocks in the clock phase of Auction 107 the opportunity to bid for specific frequency assignments. The bidding system will determine specific license assignments and any payment, above the final clock phase price, that a winning bidder will pay for the assignment.

Auction 107 will include 406 PEAs.[2] As described in the *Auction 107 Procedures Public Notice*, in the 360 PEAs that comprise only blocks subject to the Phase II deadline, there will be a single category in the clock phase, designated ABC, and thus also a single assignment category (for all 14 blocks A1–C4). In the 46 PEAs with blocks subject to the Phase I deadline,[3] there will be two categories in the clock phase (A and BC) and two assignment categories (interim and final). A winner of a Category A block will be assigned one of the five blocks subject to the Phase I deadline (A1–A5) as an interim assignment and any one of the blocks in the band (A1–C4) as a final assignment. A winner of a Category BC block will be assigned any one of the blocks in the band (A1–C4) as a final assignment and will not be assigned any block in the interim assignment.

The assignment phase is designed to promote two major goals. One of these goals is to make bidding relatively easy even though the underlying allocation problem is complex. The procedure promotes simplicity in several ways. First, to reduce the total number of bids that each bidder must make, it groups together non-top 20 PEAs within a region under certain conditions. Second, to simplify bidding strategy for bidders, it uses a *second-price* type of pricing rule that encourages bidders to bid according to their actual values for different frequencies while ensuring that there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. Third, bidding in the assignment phase is voluntary. A winner of generic blocks in the clock phase does not need to make any additional bids—or make any additional payments—in the assignment phase. This makes bidding easier in both the assignment phase and the clock phase of the auction, because bidders in the clock phase will know that they need not pay more than the prices bid in the clock phase.

A second, equally important goal is to promote efficient and intensive use of the spectrum. To achieve that, the assignment phase rules ensure that each clock phase winner is assigned contiguous frequencies within each assignment category in each PEA, even if the clock phase winner does not bid in the assignment phase. In an assignment round for a PEA with blocks subject to the Phase I deadline, there is a separate interim assignment of contiguous blocks (for clock Category A blocks) and a separate final assignment of contiguous blocks (for clock Category A and clock Category BC blocks). As a result, every winner of multiple clock Category A blocks will be assigned contiguous frequency blocks both in the interim and in the final assignment.

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[1] *See Auction of Flexible-Use Service Licenses in the 3.7–3.98 GHz Band for Next-Generation Wireless Services; Notice and Filing Requirements, Minimum Opening Bids, Upfront Payments, and Other Procedures for Auction 107*, AU Docket No. 20-25, Public Notice, DA 20-848, at 58-63, paras. 202-18 (Aug. 7, 2020) (*Auction 107 Procedures Public Notice*).

[2] The following PEAs will not be included in the auction: Honolulu, Anchorage, Kodiak, Fairbanks, Juneau, Puerto Rico, Guam-Northern Mariana Islands, U.S. Virgin Islands, American Samoa, and the Gulf of Mexico (PEA numbers 42, 212, 264, 298, 360, 412–416).

[3] The 46 PEAs subject to the Phase I deadline are all top 50 PEAs (PEAs 1-50) except for the following four PEAs: the Baltimore-Washington, Atlanta, and Denver PEAs (PEAs 5, 11 and 20) due to the need to protect Telemetry, Tracking, and Command (TT&C) sites, and the Honolulu PEA (PEA 42) because PEA 42 will not be included in Auction 107.

2 Assignment Rounds

The assignment phase consists of a series of *assignment rounds.* In each assignment round, frequencies are assigned in up to six *assignment phase markets*, with each assignment phase market consisting of either a single PEA or a group of PEAs. Winning bidders from the clock phase that have a preference for specific license frequencies submit sealed bids for those frequencies, separately for each assignment category. Once an assignment round concludes, an optimization is solved for each assignment category in each assignment phase market to assign specific frequencies to each winning bidder from the clock phase. Additional optimizations are solved to determine each bidder's assignment payment, which will be equal to or less than the bidder's bid value for the assignment.

The bidding system will determine whether to group PEAs into a single assignment phase market according to the rule detailed in Section 2.1 below.

2.1 Grouping PEAs into a Single Assignment Phase Market

A set of PEAs *P* will be grouped into one *assignment phase market* if all of the following three conditions are met:

(1) The PEAs in *P* are all in the same Regional Economic Area Grouping ("REAG")[4] and not in the top-20 PEAs;[5]
(2) Either all PEAs in *P* are not subject to the small market bidding credit cap[6] or all PEAs in *P* are subject to the small market bidding credit cap; and
(3) For each PEA in *P*, the same bidders won the same number of blocks in each clock category.

The third condition implies that all PEAs in an assignment phase market must have the same clock categories. In other words, the PEAs in an assignment phase market *either* are all subject to the Phase I deadline and thus have two clock categories (A and BC) and two assignment categories (interim and final) *or* all comprise only blocks subject to the Phase II deadline and thus have a single clock category (ABC) and a single assignment category.

Because of this grouping of PEAs, the number of assignment phase markets will be smaller than or equal to the number of PEAs in the auction.

<u>Example 1</u>: REAG Grouping

Suppose that PEA060, PEA069, and PEA077 are all in REAG 1. These PEAs are not top-20 PEAs and are not subject to the small market bidding credit cap. Each of these PEAs is not subject to the Phase I deadline and thus has a single clock category (ABC) and a single assignment category.

In each of these PEAs:

- Bidder 1 won 3 Category ABC blocks in the clock phase.
- Bidder 2 won 6 Category ABC blocks in the clock phase.

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[4] The six REAGs are: Northeast, Southeast, Great Lakes, Mississippi Valley, Central, and West.

[5] The top-20 PEAs are PEAs 1–20.

[6] PEAs that are subject to the small market bidding credit cap in Auction 107 are those PEAs with a population of 500,000 or less, which correspond to PEAs 118–211, 213–263, 265–297, 299–359, and 361–411. *See Updating Part 1 Competitive Bidding Rules et al.*, Report and Order, Order on Reconsideration of the First Report and Order, Third Order on Reconsideration of the Second Report and Order, Third Report and Order, 30 FCC Rcd 7493, 7546-47, paras. 127-28 (2015).

- Bidder 3 won 5 Category ABC blocks in the clock phase.

Then PEA060, PEA069, and PEA077 will be grouped and treated as a single combined market for the assignment phase.

Example 2: Not Possible to Group

PEA043 and PEA045 are both in REAG2 and are not in the top-20 PEAs. Neither of these PEAs is subject to the small market bidding credit cap. Each of these PEAs is subject to the Phase I deadline and thus has two clock categories (A and BC) and two assignment categories (interim and final).

The winners in these two PEAs are as follows:

In PEA043:

- Bidder 1 won 3 Category A blocks in the clock phase.
- Bidder 2 won 2 Category A blocks and 4 Category BC blocks in the clock phase.
- Bidder 3 won 5 Category BC blocks in the clock phase.

In PEA045:

- Bidder 1 won 3 Category A blocks in the clock phase.
- Bidder 2 won 2 Category A blocks and 4 Category BC blocks in the clock phase.
- Bidder 4 won 5 Category BC blocks in the clock phase.

Then, PEA043 and PEA045 will not be grouped, because they do not have identical clock phase winners.

2.2 Sequencing of Assignment Rounds

The assignment phase begins with assignment rounds for the top-20 PEAs (PEAs 1-20). The top-20 PEAs are ordered in descending order of population (pops), and bidding is conducted for a single assignment phase market per round, sequentially. Note that there is no grouping for the top-20 PEAs.

After bidding has been conducted for the top-20 PEAs, bidding is conducted simultaneously for PEAs in the six REAGs, but in descending order of PEA (or PEA group) pops within each REAG. That is, bidding may be conducted for up to six assignment phase markets at the same time, in order to speed up the assignment phase. The rounds continue until all assignment phase markets are assigned.

If an assignment phase market consists of multiple PEAs, its pops will be set to be equal to the sum of the pops of the PEAs that it comprises for purposes of determining the sequencing.

Before bidding for the assignment phase starts, the bidding system will inform bidders about which PEAs have been grouped and the sequencing of assignment rounds.

The following tables show two examples of the sequencing of assignment phase markets. In the first example, there is no grouping, that is, each assignment phase market consists of a single PEA. In the second example, some assignment phase markets consist of multiple PEAs and, as a result, there are fewer assignment rounds.

Table 1: Sequencing of assignment phase markets with no grouping

Round	PEA					
1	001					
2	002					
…	…					
20	020					
	REAG 1	REAG 2	REAG 3	REAG 4	REAG 5	REAG 6
21	041	021	023	024	028	022
22	044	029	025	030	035	026
…	…	…	…	…	…	…

Table 2: Sequencing of assignment phase markets with grouping

Round	PEA(s)					
1	001					
2	002					
…	…					
20	020					
	REAG 1	REAG 2	REAG 3	REAG 4	REAG 5	REAG 6
21	041; 044	021	023	024	028	022; 034
22	048	029; 033	025	030	035	026
…	…	…	…	…	…	…

As illustrated in the tables above, after bidding for the top-20 PEAs is finished, bidding for multiple assignment phase markets will be conducted in the same round.

3 Bidding

3.1 Bidding Options

For each of the 360 PEAs not subject to the Phase I deadline, there is a single assignment category and each bidder is assigned a specific frequency corresponding to each generic block it won in the clock phase (in Category ABC).

For each of the 46 PEAs with blocks subject to the Phase I deadline, there are two assignment categories (interim and final). In the interim assignment, a bidder is assigned frequencies for the number of generic Category A blocks it won in the clock phase. In the final assignment, a bidder is assigned frequencies for the total number of generic Category A and Category BC blocks it won in the clock phase. For example, if a bidder won one Category A block and two Category BC blocks, then it will be assigned one frequency block in the interim assignment and three frequency blocks in the final assignment.

For each assignment phase market and each assignment category, the bidding system will determine all contiguous assignment options where the number of frequency blocks is equal to the number of blocks that the winner is to be assigned in that category and assignment phase market. This set is referred to as the *bidding options* of the bidder. Note that the bidding options of a bidder do not depend on the clock phase winnings of other bidders.

Example 3: Consider an assignment phase market that consists of PEAs not subject to the Phase I deadline. A bidder won 4 Category ABC blocks in each of these PEAs. Then, the bidder will have the following 11 bidding options: A1-A4, A2-A5, A3-B1, A4-B2, A5-B3, B1-B4, B2-B5, B3-C1, B4-C2, B5-C3, and C1-C4.

Example 4: Consider an assignment phase market that consists of PEAs subject to the Phase I deadline. A bidder won 1 Category A block for each of those PEAs and did not win any Category BC blocks for those PEAs. Then, the bidder will have a set of bidding options for each assignment category.

- For the interim assignment, the bidder will be assigned 1 block and will have the following 5 bidding options: A1, A2, A3, A4, and A5.
- For the final assignment, the bidder will be assigned 1 block and will have the following 14 bidding options: A1, A2, . . ., C4.

Example 5: Consider an assignment phase market that consists of PEAs subject to the Phase I deadline. A bidder won 4 Category BC blocks for each of these PEAs and did not win any Category A blocks for those PEAs. Then, the bidder has the following 11 bidding options: A1-A4, A2-A5, A3-B1, A4-B2, A5-B3, B1-B4, B2-B5, B3-C1, B4-C2, B5-C3, and C1-C4.

Example 6: Consider an assignment phase market that consists of PEAs subject to the Phase I deadline. A bidder won 2 Category A blocks and 4 Category BC blocks for each of these PEAs. Then, the bidder will have a set of bidding options for each assignment category.

- For the interim assignment, the bidder will be assigned 2 blocks and will have the following 4 bidding options: A1A2, A2A3, A3A4, and A4A5.
- For the final assignment, the bidder will be assigned 6 blocks and will have the following 9 bidding options: A1-B1, A2-B2, A3-B3, A4-B4, A5-B5, B1-C1, B2-C2, B3-C3, and B4-C4.

The bidder can bid on any of its bidding options. Note that in certain instances, however, some of the bidding options cannot be assigned. For instance, if another bidder won 3 Category A blocks and thus is being assigned 3 blocks in the interim assignment, the bidder in Example 6 above cannot be assigned A2A3 or A3A4 because, if it were, it would not be possible to assign contiguous spectrum to the other bidder. The bidding options of a bidder are not limited only to the options that can be won by the bidder, because limiting the bidding options in that way may permit a bidder to infer the clock phase winnings of other bidders.

3.2 Automatic Assignments and Pre-Assignments

If a bidder has only one bidding option available in an assignment phase market for a given assignment category, the bidder will be automatically assigned the frequencies in this option and will not be able to submit bids for this assignment phase market and category. For example, a bidder that won 5 Category A blocks in a PEA will be automatically assigned A1-A5 for the interim assignment.

If all winners in a PEA have only one bidding option available to each of them, then all frequency blocks in that PEA are pre-assigned to winners and there will not be a round held for that PEA. Similarly, if a PEA has no winners from the clock phase (all blocks remain FCC-held), there will not be a round held for that PEA. In the context of Auction 107, a PEA will be pre-assigned if *either* all 14 blocks were won by a single bidder *or* all 14 PEAs remain FCC-held.

3.3 Bidding Rules

A bidder may specify a bid value for each of its bidding options for an assignment phase market. The bidder bids for a bidding option by specifying a bid amount for that option. The bid amount must be non-negative, must be a multiple of $10, and cannot exceed $999,999,990.

A bidder that does not have clock phase winnings in a PEA will not have any bidding options in the corresponding assignment phase market and thus cannot submit bids for that market.

If an assignment phase market is a group of PEAs, then each bidder has the same clock phase winnings in each of those PEAs (because of the grouping rule described in Section 2.1). By specifying a bid value for a bidding option, the bidder indicates the maximum amount that it is willing to pay to be assigned that option in all those PEAs. A bidder will not be able to bid for different frequency assignments in the various PEAs within a group.

A clock phase winner is not required to bid in the assignment phase. In particular, such a bidder may not wish to bid if it is indifferent among all assignments that it may get. The bidding system will consider a bid value of zero for any set of frequency blocks for which a bidder submits no bid.

3.4 Winning Assignments and Payments

After bidding in an assignment round concludes, the bids are processed to determine the winning assignments and the payments for that round. For each assignment phase market of the round, each bidder is then informed about its winning assignment and its assignment payment for each assignment category in that assignment phase market. This information is disclosed to the bidder before the next assignment round starts.

4 Assignment Determination

For a given assignment category in a given assignment phase market, an assignment is *feasible* if:

(1) Each bidder is assigned one of its bidding options; and

(2) Any FCC-held frequency blocks are contiguous.

For the 46 PEAs subject to the Phase I deadline, the winning assignment is determined separately for the interim assignment and for the final assignment by maximizing the sum of bid values across all bids for the corresponding assignment category. Ties, if any, are broken by including pseudorandom numbers in an optimization.

Specifically, the assignment determination is done by solving two optimization problems for each assignment category in each assignment phase market. The first optimization problem finds the maximum sum of bid values among all feasible assignments. The bidding system then solves another optimization problem using randomly generated numbers to break ties, if any. The solution to the latter optimization is selected as the winning assignment.

To mathematically formulate the assignment determination, the following notation is used:

- N denotes the set of bidders in that assignment phase market and assignment category, that is, the set of clock phase winners to be assigned blocks in this particular assignment phase market and this particular assignment category.
- The FCC is referred to as bidder 0, and $N \cup \{0\}$ is used to denote the set of bidders and the FCC.
- K denotes the set of frequency blocks for the category. For the interim assignment, K consists of frequency blocks A1-A5. In every other case, K consists of frequency blocks A1-C4.

- S denotes a set of frequency blocks. For each block k, S_k denotes the indicator variable of whether block k is in set S. That is, $S_k = 1$ if $k \in S$, and $S_k = 0$ if $k \notin S$.
- m_i is the number of frequency blocks per PEA to be assigned to bidder i in the assignment phase market and assignment category of interest.
- m_0 is the number of FCC-held frequency blocks per PEA in the assignment phase market and assignment category of interest.
- F_i denotes the set of bidding options for bidder i. That is, F_i will consist of all sets $S \subseteq K$ with m_i contiguous frequency blocks (see Section 3 for examples).
- F_0 consists of all sets $S \subseteq K$ with m_0 contiguous frequency blocks. This set gives the possible assignments for any FCC-held blocks.
- $b_i(S)$ denotes the bid value of bidder i for set $S \in F_i$.
- b denotes the set of bid values.

Variable Definition:

$X_i(S)$ is a binary decision variable which has a value of 1 if exactly the frequency blocks of set S are assigned to bidder i, and 0 otherwise. This variable is defined for all $i \in N \cup \{0\}$. Thus, $X_0(S) = 1$ if the set of frequency blocks assigned to the FCC is S.

4.1 Maximum Sum of Bid Values

$$r(b) = max \sum_{i \in N} \sum_{S \in F_i} b_i(S) \cdot X_i(S)$$

Subject to:

$$\sum_{i \in N \cup \{0\}} \sum_{S \in F_i} S_k \cdot X_i(S) = 1 \qquad \forall k \in K \qquad (1)$$

$$\sum_{S \in F_i} X_i(S) = 1 \qquad \forall i \in N \cup \{0\} \qquad (2)$$

$$X_i(S) \in \{0,1\} \qquad \forall i \in N \cup \{0\}, \forall S \in F_i \qquad (3)$$

The objective function is equal to the sum of bid values of an assignment, across all bidders.

Explanation of Constraints:

- Constraint (1) ensures that each frequency block is assigned exactly once, either to one of the bidders or to the FCC.
- Constraint (2) ensures that each bidder is assigned exactly one of its bidding options and that the set of frequency blocks assigned to the FCC is contiguous.
- Constraint (3) states that each decision variable $X_i(S)$ can be either equal to 0 or 1.

4.2 Tie-breaking

For every set S and every bidder $i \in N$, the bidding system generates a pseudorandom number $\xi_i(S)$ drawn uniformly at random from the set $\{1,2, \ldots, 10^7\}$. The bidding system then solves an optimization problem to find the assignment that maximizes the sum of pseudorandom numbers among all assignments that satisfy constraints (1) through (3) of Section 4.1 such that the sum of bid values is equal to $r(b)$. In particular, the optimization problem is formulated as follows:

$$max \sum_{i \in N} \sum_{S \in F_i} \xi_i(S) \cdot X_i(S)$$

Subject to:

$\sum_{i \in N \cup \{0\}} \sum_{S \in F_i} S_k \cdot X_i(S) = 1$	$\forall k \in K$	(1)
$\sum_{S \in F_i} X_i(S) = 1$	$\forall i \in N \cup \{0\}$	(2)
$X_i(S) \in \{0,1\}$	$\forall i \in N \cup \{0\}, \forall S \in F_i$	(3)
$\sum_{i \in N} \sum_{S \in F_i} b_i(S) \cdot X_i(S) \geq r(b)$		(4)

Constraints (1) through (3) are the same as in the optimization of Section 4.1.

Explanation of New Constraint:

Constraint (4) states that the sum of bid values must be greater than or equal to the result of the optimization of Section 4.1.

5 Assignment Payment Determination

A bidder's assignment payment for the set of frequency blocks it is assigned for an assignment category in an assignment phase market is an additional payment amount above its clock phase payments. If a bidder did not bid (or submitted a bid of zero) for the set of frequency blocks that it is assigned, then no additional calculation is necessary, and the bidder will not have any additional assignment payment for that assignment category in that assignment phase market. If, on the other hand, the bidder submitted a positive bid for the winning assignment, then the bidding system will calculate a type of 'second-price' assignment payment.[7]

The bidding system will apply bidder-optimal core prices and will use the "nearest Vickrey" approach to determine the assignment payments. In some cases, the second price (Vickrey price) may not be high enough to ensure that no bidder or group of bidders is willing to pay more for an alternative feasible assignment, and so an additional payment above Vickrey prices may be required. In the event that such a payment is required, the calculation of the additional payment to be paid by each bidder will be weighted based on the number of blocks being assigned to the bidder for that assignment category in that assignment phase market.

The assignment payments will satisfy the following conditions:

First condition: Each assignment payment must be positive or zero and not more than the dollar amount of the winning assignment phase bid.

Second condition: The set of assignment payments must be sufficiently high that there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. If there is only one set of assignment payments that satisfies the first two conditions, this determines the assignment payments.

Third condition: If there is more than one set of assignment payments that fulfill the first and second conditions, the set(s) of assignment payments minimizing the sum of assignment payments across all

[7] In some cases, this may also be zero.

bidders is (are) selected. If there is only one set of assignment payments that satisfies these three conditions, this determines the assignment payments.

Fourth condition: If there is more than one set of assignment payments that satisfies the first three conditions, the set of assignment payments that minimizes the weighted sum of squares of differences between the assignment payments and the Vickrey prices will be selected. The weighting is relative to the number of blocks assigned to the bidder in the assignment category and assignment phase market. This approach for selecting among sets of assignment payments that minimize the sum of assignment payments across bidders is referred to as the "nearest Vickrey" approach.

Section 5.1 describes how the Vickrey prices are calculated. Section 5.2 describes how payments are adjusted (if needed) to ensure that there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. Section 5.3 provides an example.

5.1 Vickrey Price Calculation

For each bidder, the bidding system will determine the Vickrey price by re-solving the optimization problem of Section 4.1, but setting all bids of the bidder to zero while keeping the bids of every other bidder unchanged from the prior optimization, and calculating a hypothetical maximum sum of bid values from that optimization. The difference between the maximum sum of bid values associated with the actual optimization and the hypothetical maximum sum of bid values that would occur had that bidder provided all bids of zero will indicate the amount by which the bidder's winning bid amount exceeded the minimum amount it would have needed to bid to ensure the same winning assignment. The Vickrey price is calculated by subtracting that amount from the bidder's actual bid amount.

Specifically, let $r(b)$ denote the maximum value attained by solving the optimization problem of Section 4.1 where the set of bid values is b. Let b_i^* be the bid amount of bidder i for the bidding option that it is assigned.

The Vickrey price of bidder i for a given assignment category in a given assignment phase market is:

$$p_i^{Vickrey} = b_i^* - (\, r(b) - r(b_{i\to 0})\,)$$

where $b_{i\to 0}$ represents the set of bid values where the bid values of all bids of bidder i are set to zero (and the bid values of every other bidder are not changed).

5.2 Core Adjustments

An extra payment beyond the Vickrey prices is sometimes required in order to satisfy the second condition, which requires that the set of assignment payments is sufficiently high that there is no bidder or group of bidders prepared to pay more for an alternative feasible assignment. A bidder or group of bidders willing to pay more for an alternative feasible assignment is referred to as a blocking coalition of bidders. The group which is willing to pay the most forms the first blocking coalition. A blocking coalition is unblocked by increasing the assignment payments while ensuring that assignment payments are increased no more than necessary and that each bidder's assignment payment is less than or equal to the corresponding bid amount. After adjusting the assignment payments to unblock the first blocking coalition, additional blocking coalitions may exist that are each unblocked by again increasing the assignment payments until there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. Each bidder's assignment payment is guaranteed to be at least its Vickrey price and no more than its bid amount for its assignment.

Assignment payments can be calculated iteratively via a core adjustment process. This process operates by starting with Vickrey prices and then by iteratively adjusting assignment payments until there is no bidder or group of bidders willing to pay more for an alternative feasible assignment. It does so by gathering pricing constraints from each blocking coalition and then satisfying the pricing constraints by selecting assignment payments which minimize the distance, weighted by the number of blocks being assigned to each bidder in the given assignment category and assignment phase market, from Vickrey prices.

To mathematically formulate the core adjustment calculations, the following notation is used in addition to the notation of Section 4:

- b_i^* is the bid amount of bidder i for the bidding option that it is assigned (in the original assignment determination problem).
- p_i^n is the payment of bidder i for core adjustment in iteration n.
- C^n denotes the blocking coalition for iteration n.

In the first iteration, the payment of bidder i is set equal to the Vickrey price of bidder i, that is, $p_i^1 = p_i^{Vickrey}$.

Given a set of assignment payments for iteration n, calculate a reduced bid for each bidder and each of its bidding options by deducting the current surplus of the bidder $(b_i^* - p_i^n)$ from the corresponding bid amount. Specifically, in iteration n, the reduced bid of bidder i for a bidding option $S \in F_i$ is:

$$b_i^n(S) = \max\{b_i(S) - (b_i^* - p_i^n), 0\}$$

Let C^n be the set of bidders with strictly positive bid amounts for the options they are assigned in the solution to the assignment determination problem for the set of reduced bids b^n in the optimization problem of Section 4.1. These bidders form the potential blocking coalition for iteration n. Among all potential blocking coalitions for iteration n, C^n is the one with the highest value, that is, $r(b^n)$.

If the value of the potential blocking coalition for iteration n does not exceed the sum of assignment payments for iteration n (that is, if $r(b^n) \leq \sum_{i \in N} p_i^n$), then there is no blocking coalition for iteration n, and p_i^n represents the assignment payment for bidder i. In this case, the assignment payment for bidder i is determined by rounding p_i^n up to the nearest integer, and no further calculations are required.

Otherwise, bidders in C^n do form a blocking coalition for iteration n, and the bidding system will calculate the updated set of assignment payments p_i^{n+1} as described below.

The bidding system will first calculate the minimum sum of assignment payments required to unblock all coalitions as of iteration n. This is done by solving the following optimization problem:

Minimize Sum of Assignment Payments (for third condition)

$$\mu^n = \min \sum_{i \in N} p_i$$

Subject to:

$\sum_{i \in N \setminus C^k} p_i \geq r(b^k) - \sum_{i \in C^k} p_i^k$	$\forall k \in \{1, \dots, n\}$	(1)
$p_i \geq p_i^{Vickrey}$	$\forall i \in N$	(2)
$p_i \leq b_i^*$	$\forall i \in N$	(3)

Explanation of Constraints:

- Constraint (1) ensures that all coalitions are unblocked. In other words, for the blocking coalition of iteration k, the sum of assignment payments by bidders not in the coalition must be greater than or equal to the value of the coalition under the set of reduced bids for iteration k less the total iteration k payments of bidders in that coalition. There is one such constraint for each iteration.

- Constraint (2) requires that the price paid by each bidder be greater than or equal to the bidder's Vickrey price.
- Constraint (3) requires that the price paid by each bidder be less than or equal to the bidder's bid amount for its winning assignment.

The bidding system will then update the assignment payments of bidders such that they sum up to μ^n (the third condition).

If there is more than one set of assignment payments that sum up to μ^n, the set of assignment payments that minimizes the weighted sum of squares of differences between the assignment payments and the Vickrey prices will be selected. The weighting is relative to the number of blocks that the bidder is being assigned in that assignment category in that assignment phase market (the fourth condition).

The updated assignment payments p_i^{n+1} are calculated as the optimal solution to:

Minimize Distance Between Assignment Payments and Vickrey Prices (for fourth condition)

$$\min \sum_{i \in N} \frac{(p_i^{n+1} - p_i^{Vickrey})^2}{m_i}$$

Subject to:

$\sum_{i \in N \setminus C^k} p_i^{n+1} \geq r(b^k) - \sum_{i \in C^k} p_i^k$	$\forall k \in \{1, \dots, n\}$	(1)
$p_i^{n+1} \geq p_i^{Vickrey}$	$\forall i \in N$	(2)
$p_i^{n+1} \leq b_i^*$	$\forall i \in N$	(3)
$\sum_{i \in N} p_i^{n+1} = \mu^n$		(4)

This quadratic problem minimizes the weighted sum of squares of differences between the updated assignment payments p_i^{n+1} and the Vickrey prices $p_i^{Vickrey}$, weighted by the number of blocks (m_i) won by each bidder.

Constraints (1) through (3) are the same as in the previous optimization.

Explanation of New Constraint:

Constraint (4) ensures that the sum of the updated payments is equal to the minimum amount required to unblock all of the coalitions up to iteration n.

5.3 Assignment Payment Calculation Example

This section provides an example to illustrate how Vickrey prices and core adjustments are calculated in order to determine the assignment payments. In this example, in a PEA subject to the Phase I deadline, Bidder 1 won 1 Category A block, and Bidders 2 and 3 won 2 Category A blocks each. Thus, for the interim assignment, Bidder 1 will be assigned 1 block and Bidders 2 and 3 will be assigned 2 blocks each.

The bids of each bidder for the interim assignment are shown in Table 3. The positive bid amounts are summarized below:

- Bidder 1 bids \$1,000 on A5.
- Bidder 2 bids \$2,000 on A2A3.

- Bidder 3 bids $3,000 on A4A5.

Assignment Determination: The sum of bid values is maximized when Bidder 1 is assigned A1, Bidder 2 is assigned A2A3, and Bidder 3 is assigned A4A5. The value of this assignment is equal to $5,000.

Vickrey Prices: In this example, all Vickrey prices are $0 even though Bidder 3 is assigned a block for which Bidder 1 bid $1,000.

- The Vickrey price of Bidder 1 is equal to $0, because its bid value for its assignment is $0.
- To calculate the Vickrey price of Bidder 2, the bidding system solves the assignment determination optimization problem with all of the bids of Bidder 2 set to $0. The optimal value of this optimization problem is $3,000, because Bidder 3 would be assigned A4A5 (with a value of $3,000) and the remaining blocks would be assigned to Bidders 1 and 2 (with a value of $0). Thus, the Vickrey discount for Bidder 2 is equal to $5,000-$3,000 = $2,000. The Vickrey price of Bidder 2 is then calculated as its bid amount for its assignment ($2,000) less its Vickrey discount ($2,000) and is therefore equal to $0.
- To calculate the Vickrey price of Bidder 3, the bidding system solves the assignment determination optimization problem with all of the bids of Bidder 3 set to $0. The optimal value of this optimization problem is $2,000, because Bidder 1 would be assigned A1 (with a value of $0), Bidder 2 would be assigned A2A3 (with a value of $2,000) and Bidder 3 would be assigned blocks A4A5 (with a value of $0). Note that it is not feasible to assign A5 to Bidder 1 and A2A3 to Bidder 2, because then it would not be possible to assign contiguous spectrum to Bidder 3. Thus, the Vickrey discount for Bidder 3 is equal to $5,000-$2,000 = $3,000. The Vickrey price of Bidder 3 is then calculated as its bid amount for its assignment ($3,000) less its Vickrey discount ($3,000) and is therefore equal to $0.

<u>Iteration 1</u>. The next step is to determine whether there is a blocking coalition for iteration 1. To do this, the bidding system first calculates the surplus of each bidder for the first iteration as the bidder's bid amount for its assignment less its Vickrey price. The surplus is $0 for Bidder 1, $2,000 for Bidder 2, and $3,000 for Bidder 3. The bidding system then calculates a set of reduced bids for each bidder by subtracting the bidder's surplus from its actual bid amount for each bidding option. If the result for a bidding option is negative, the reduced bid amount is set to be equal to $0. The reduced bid amounts for the first iteration, b^1, are shown in Table 3. The bids of Bidder 1 are not reduced because Bidder 1 derives $0 surplus. The bids of Bidder 2 are reduced by up to $2,000 while ensuring that all bid values are non-negative. Similarly, the bids of Bidder 3 are reduced by up to $3,000 while ensuring that all bid values are non-negative.

The assignment determination problem is solved with the set of reduced bids for iteration 1. There is a blocking coalition (consisting of Bidder 1) with value $1,000. That is, Bidder 1 is willing to pay up to $1,000 more than the sum of Vickrey prices (which is $0) in order to be assigned A5. The bidding system then calculates that the minimum sum of assignment payments to unblock this coalition is $\mu^1 = 1{,}000$. Because in this example Bidders 2 and 3 are being assigned the same number of blocks, the assignment payment of each of those bidders is incremented by the same amount, namely, by $500. Thus, the assignment payments for iteration 2 are: $p_1^2 = 0$ and $p_2^2 = p_3^2 = 500$.

<u>Iteration 2</u>. The bidding system checks whether there exists another blocking coalition by solving the assignment determination problem for the reduced set of bids b^2 shown in Table 3. The maximum sum of bids is equal to $1,000, which does not exceed the sum of assignment payments for this iteration. Thus, there does not exist a blocking coalition in iteration 2. This implies that the assignment payments are equal to p^2, that is, $0 for Bidder 1, $500 for Bidder 2, and $500 for Bidder 3.

Table 3: Assignment Payment Calculation Example

Bidders	*Bidder 1 (1 block)*	*Bidder 2 (2 blocks)*	*Bidder 3 (2 blocks)*
Bids	**A1: $0**	A1A2: $0	A1A2: $0
	A2: $0	**A2A3: $2,000**	A2A3: $0
	A3: $0	A3A4: $0	A3A4: $0
	A4: $0	A4A5: $0	**A4A5: $3,000**
	A5: $1,000		
Vickrey prices (p^1)	$0	$0	$0
Bidder Surplus at p^1	$0	$2,000	$3,000
Reduced Bids Iteration 1 (b^1)	A1: $0	A1A2: $0	**A1A2: $0**
	A2: $0	A2A3: $0	A2A3: $0
	A3: $0	**A3A4: $0**	A3A4: $0
Coalition	A4: $0	A4A5: $0	A4A5: $0
$C^1 = \{1\}$	**A5: $1,000**		
Adjusted payments p^2 (Assignment payments)	$0	$500	$500
Bidder Surplus at p^2	$0	$1,500	$2,500
Reduced Bids Iteration 2 (b^2)	**A1: $0**	A1A2: $0	A1A2: $0
	A2: $0	**A2A3: $500**	A2A3: $0
	A3: $0	A3A4: $0	A3A4: $0
No blocking	A4: $0	A4A5: $0	**A4A5: $500**
coalition	A5: $1,000		

6 Final Auction Payment

This section describes how a bidder's final auction payment is calculated at the conclusion of the assignment phase.

This section uses the following notation:

- $d_{T,i,\{c,j\}}$ denotes the processed demand of bidder i for clock category c in PEA j after the final clock round. This is the number of blocks that the bidder won in clock category c in PEA j.
- $p_{T,\{c,j\}}$ denotes the posted price for clock category c in PEA j after the final clock round. This is the final clock phase price.
- R denotes the set of clock phase products, that is, all pairings of a PEA and a clock category.
- $AP_{i,k}$ denotes the assignment payment of bidder i in assignment phase market k. For an assignment phase market k with two assignment categories (interim and final), $AP_{i,k} = AP_{i,k,Interim} + AP_{i,k,Final}$, where $AP_{i,k,Interim}$ is the assignment payment for the interim assignment and $AP_{i,k,Final}$ is the assignment payment for the final assignment.
- AM denotes the set of assignment phase markets.

- SM denotes the set of assignment phase markets that consist of PEAs that are small markets.[8]
- $APM(k)$ denotes the set of PEAs in assignment phase market k.
- BC_i denotes the bidding credit percentage of bidder i.
- FGP_i is the final gross payment of bidder i.
- FD_i is the final discount of bidder i.
- $GP_{i,k}$ denotes the gross payment of bidder i for assignment phase market k. This is equal to the sum of the final clock phase prices for all licenses in the bidder's assignment *and* the bidder's assignment payment(s) in assignment phase market k.

  For an assignment phase market k with a single assignment category—and thus a single clock category ABC—the gross payment of bidder i is:

$$GP_{i,k} = \sum_{j \in APM(k)} d_{T,i,\{ABC,j\}} \cdot p_{T,\{ABC,j\}} + AP_{i,k}$$

  For an assignment phase market k with two assignment categories (interim and final)—and thus two clock categories (A and BC)—the gross payment of bidder i is:

$$GP_{i,k} = \sum_{j \in APM(k)} \left(d_{T,i,\{A,j\}} \cdot p_{T,\{A,j\}} + d_{T,i,\{BC,j\}} \cdot p_{T,\{BC,j\}} \right) + AP_{i,k,Interim} + AP_{i,k,Final}$$

When all assignment rounds have completed, a bidder's final gross payment is determined by calculating the sum of the final clock phase prices across all licenses that it won and its assignment payments across all assignment phase markets. Equivalently, the final gross payment of bidder i is equal to the sum of its gross payments across all assignment phase markets and categories:

$$FGP_i = \sum_{k \in AM} GP_{i,k}$$

For a bidder that does not qualify for a bidding credit, the *final auction payment* is equal to its final gross payment.

For a bidder that qualifies for a bidding credit, the *final auction payment* is equal to its final gross payment minus its final discount, that is, $FGP_i - FD_i$.

If bidder i qualifies for the rural service provider bidding credit, then its final discount is:

$$FD_i = \min\{\$10 \text{ million}, BC_i \cdot FGP_i\}$$

If bidder i qualifies for the small business bidding credit, then its final discount is:

$$FD_i = \min\left\{\$25 \text{ million}, BC_i \cdot \sum_{k \in AM \setminus SM} GP_{i,k} + \min\left\{\$10 \text{ million}, BC_i \cdot \sum_{k \in SM} GP_{i,k}\right\}\right\}$$

All the discount calculations described above will be rounded to the nearest dollar. Rounding will only be done at the very end of a given calculation, that is, after all summations, multiplications, and minimizations in a formula have been performed.

[8] Note that according to the rules for grouping PEAs into a single assignment phase market described in Section 2.1, *either* all PEAs in an assignment phase market are subject to the small market bidding credit cap *or* none of the PEAs in that assignment phase market are subject to the small markets bidding credit cap.

7 Calculations for Payment Information During Assignment Phase

While winning bidders will be expected to pay the final auction payment set forth above, the bidding system will show each bidder a running estimate of its auction payment obligations during the assignment phase. After each assignment round, each bidder will be shown its current total payment. A bidder that qualifies for a bidding credit will also be shown its current discount and total net payment.

This section uses the notation of Section 6.

7.1 Total Payment

The *total payment* of bidder i when A is the set of assignment phase markets for which an assignment has been processed is calculated as:

$$TP_i(A) = \sum_{r \in R} d_{T,i,r} \cdot p_{T,r} + \sum_{k \in A} AP_{i,k}$$

The first term, which represents the clock phase payment, is the product of the final clock phase price and the number of blocks won by the bidder, summed over all products. The second term is the sum of the bidder's assignment payments across all assignment phase markets that have been assigned so far. When an assignment has been processed for all assignment phase markets, the bidder's total payment is equal to its final gross payment.

7.2 Bidding Credit Discounts

All the discount calculations described in this section will be rounded to the nearest dollar. Rounding will only be done at the very end of a given calculation, that is, after all summations, multiplications, and minimizations in a formula have been performed.

7.2.1 Rural Service Provider Bidding Credit

Suppose that bidder i qualifies for the rural service provider bidding credit, and A is the set of assignment phase markets for which an assignment has been processed.

The current *uncapped discount* of bidder i is calculated as:

$$BC_i \cdot TP_i(A)$$

The current *discount* of bidder i is calculated as:

$$min\{\$10 \text{ million}, BC_i \cdot TP_i(A)\}$$

That is, the bidder's total payment is multiplied by its bidding credit percentage and capped at $10 million.

7.2.2 Small Business Bidding Credit

In this section, $TP_{i,SM}(A)$ is used to denote the part of the total payment of bidder i that relates to PEAs subject to the small market bidding credit cap, and $TP_{i,NSM}(A)$ is used to denote the part of the total payment that relates to PEAs not subject to the small market bidding credit cap.

Suppose that bidder i qualifies for the small business bidding credit, and A is the set of assignment phase markets for which an assignment has been processed.

The current *uncapped discount for small markets* only is:

$$BC_i \cdot TP_{i,SM}(A)$$

The current *uncapped discount* (across all markets) is:

$$BC_i \cdot TP_i(A)$$

The current *discount* (across all markets) is:

$$min\{\$25 \text{ million}, \; BC_i \cdot TP_{i,NSM}(A) + min\{\$10 \text{ million}, BC_i \cdot TP_{i,SM}(A)\}\}$$

This calculation first caps the discount from small markets at $10 million, then adds the discount from all other markets and caps the total at $25 million.

7.3 Total Net Payment

The *total net payment* is equal to the bidder's total payment minus its discount. Once all assignment rounds have been processed, a bidder's total net payment is equal to its final auction payment.

8 Pairing Interim and Final Assignments

For a PEA that is one of the 46 PEAs subject to the Phase I deadline and thus has two assignment categories (interim and final), a winner is assigned specific frequencies for each block won in clock category A (which includes an interim and a final assignment) and for each block won in clock category BC (which includes only a final assignment).[9]

To provide each winning bidder with frequency-specific licenses based on its interim and final assignments, the bidding system will pair interim and final assignments as follows: First, any blocks that appear both in a bidder's interim and final assignment will be matched together. Then, considering all remaining blocks in the bidder's interim assignment (from lowest to highest), each will be paired with the lowest available block in the bidder's final assignment.

Example 7: In a PEA subject to the Phase I deadline, a bidder won two Category A blocks and one Category BC block. The bidder is assigned blocks A2 and A3 in the interim assignment and blocks A5, B1, and B2 in the final assignment. The interim assignment for A2 will be paired with the final assignment for A5, the interim assignment for A3 with the final assignment for B1, and the bidder will have a single final assignment for B2.

Example 8: In a PEA subject to the Phase I deadline, a bidder won two Category A blocks and one Category BC block. The bidder is assigned blocks A2 and A3 in the interim assignment and blocks A3, A4, and A5 in the final assignment. The interim assignment for A3 will be paired with the final assignment for A3. The interim assignment for A2 will be paired with the final assignment for A4, and the bidder will have a single final assignment for A5.

9 Per-License Price Calculations

While final auction payments for winning bidders will be calculated as in Section 6 above, with bidding credit caps and assignment payments applying on an aggregate basis rather than for individual license

[9] As noted in the *Auction 107 Procedures Public Notice*, a winner of a block or blocks in clock Category A will be awarded a paired license authorization if its interim and final frequency assignments are for different frequencies, rather than a single license authorization. *See Auction 107 Procedures Public Notice* at 63-64, paras. 219-22.

authorizations, the bidding system will also calculate a "per-license" price for each license authorization, where a license authorization in this context may refer to a paired combination of an interim and a final assignment for different frequency blocks. Such individual prices may be needed in the event that a licensee subsequently incurs license-specific obligations, such as unjust enrichment payments.

After the assignment phase, the bidding system will determine a net and gross price for each license authorization (single or paired) that was won by a bidder by apportioning assignment payments and bidding credit discounts (only applicable for the net price) across all the licenses that the bidder won. To calculate the gross price for a given license in an assignment phase market with only blocks subject to the Phase II deadline (clock category ABC), the bidding system will apportion the assignment payment for the corresponding assignment phase market to the licenses that the bidder is assigned in that market in proportion to the final clock phase price of those blocks. In the case of an assignment phase market with two assignment categories (interim and final), the assignment payment for the final assignment will be apportioned to all the licenses that the bidder is assigned in that market, whereas the assignment payment for the interim assignment will be apportioned only to license authorizations that include an interim assignment for an A block.

To calculate the net price, the bidding system will first apportion any applicable bidding credit discounts to each assignment phase market in proportion to the gross payment for that market. Then, for each assignment phase market, the bidding system will apportion the assignment payment and the discount to licenses in proportion to the final clock phase prices of the licenses that the bidder is assigned for that market.

This section uses the same notation as Section 6.

9.1 Apportioning Discounts to Each Assignment Phase Market

This section describes how to apportion the bidder's final bidding credit discount across assignment phase markets. Section 6 describes how the bidder's final discount (FD_i) is calculated. Let $D_{i,k}$ denote the discount that is apportioned to assignment phase market k.

If bidder i does not qualify for any bidding credit discount (and thus $FD_i = 0$), then $D_{i,k} = 0$ for all assignment phase markets.

A bidder i that qualifies for the small business bidding credit is considered to have exceeded the small markets bidding credit cap if $BC_i \cdot \sum_{k \in SM} GP_{i,k}$ rounded to the nearest integer is greater than $10 million.

If bidder i qualifies for the rural service provider bidding credit *or* if the bidder qualifies for the small business bidding credit and did not exceed the small markets cap, then:

$$D_{i,k} = \frac{GP_{i,k}}{FGP_i} \cdot FD_i$$

That is, the final discount is apportioned to assignment phase markets proportionally to the bidder's gross payment in each assignment phase market. Recall that the final gross payment, FGP_i, is equal to the sum of gross payments across all assignment phase markets (see Section 6).

For each assignment phase market, the calculation is rounded down to the nearest dollar. The slack due to rounding down is then distributed (one dollar at a time) to assignment phase markets based on ascending order of gross payments. Ties are broken based on ascending lexicographic order of assignment phase market ID. The assignment phase market ID is defined as the PEA number of the lowest-numbered PEA in the assignment phase market.

If bidder i qualifies for the small business bidding credit and it exceeded the small markets bidding credit cap, then:

- If $k \in SM$,

$$D_{i,k} = \frac{GP_{i,k}}{\sum_{k' \in SM} GP_{i,k'}} \cdot (\$10 \text{ million})$$

- If $k \notin SM$,

$$D_{i,k} = \frac{GP_{i,k}}{\sum_{k' \in AM \setminus SM} GP_{i,k'}} \cdot (FD_i - \$10 \text{ million})$$

That is, the $10 million discount that applies to small markets is apportioned to assignment phase markets that consist of PEAs subject to the small market bidding credit cap proportionally to the bidder's gross payments. The remaining discount ($i.e.$, $FD_i - \$10$ million) is apportioned among the assignment phase markets that consist of PEAs not subject to the small market bidding credit cap.

For each assignment phase market, the calculation is rounded down to the nearest dollar. The slack due to rounding down is then distributed (one dollar at a time) to assignment phase markets based on ascending order of gross payments. Ties are broken based on ascending lexicographic order of assignment phase market ID.

In the case of a small business that exceeded the small markets bidding credit cap, the apportioning of discounts and the distribution of any slack is done separately for the small markets and for the non-small markets.

9.2 Calculation of Gross Per-License Prices

Gross per-license prices are calculated by apportioning assignment phase payments to licenses in proportion to final clock phase prices.

Suppose that the bidder has been assigned a license in PEA j. Let k be the assignment phase market in which PEA j was assigned, that is, $j \in APM(k)$.

If PEA j is one of the 360 PEAs not subject to the Phase I deadline and thus has a single assignment category and a single license authorization for each block won in the clock phase, the gross per-license price of a license in the PEA is determined by the following formula:

$$p_{T,i,\{ABC,j\}} + \frac{p_{T,i,\{ABC,j\}}}{\sum_{j' \in APM(k)} d_{T,i,\{ABC,j'\}} \cdot p_{T,i,\{ABC,j'\}}} \cdot AP_{i,k}$$

That is, the assignment phase payment is apportioned to the license authorizations in that assignment phase market in proportion to the final clock phase price of each block. Note that if the bidder's assignment payment in the assignment phase market is zero, then the gross per-license price of each license authorization it is assigned in that market is simply the final clock phase price.

If PEA j is one of the 46 PEAs subject to the Phase I deadline and thus has two assignment categories (interim and final), a winner is assigned a separate license authorization for each block won in clock category A (which includes an interim and a final assignment) and for each block won in clock category BC (which includes only a final assignment).

The gross per-license price of a license authorization corresponding to a clock category A block in PEA j is determined by the following formula:

$$p_{T,i,\{A,j\}} + \frac{p_{T,i,\{A,j\}}}{\sum_{j' \in APM(k)} d_{T,i,\{A,j'\}} \cdot p_{T,i,\{A,j'\}}} \cdot AP_{i,k,Interim} + \frac{p_{T,i,\{A,j\}}}{\sum_{j' \in APM(k)} (d_{T,i,\{A,j'\}} \cdot p_{T,i,\{A,j'\}} + d_{T,i,\{BC,j'\}} \cdot p_{T,i,\{BC,j'\}})} \cdot AP_{i,k,Final}$$

The gross per-license price of a license corresponding to a clock category BC block in PEA j is determined by the following formula:

$$p_{T,i,\{BC,j\}} + \frac{p_{T,i,\{BC,j\}}}{\sum_{j' \in APM(k)} (d_{T,i,\{A,j'\}} \cdot p_{T,i,\{A,j'\}} + d_{T,i,\{BC,j'\}} \cdot p_{T,i,\{BC,j'\}})} \cdot AP_{i,k,Final}$$

That is, the assignment phase payment for the interim assignment is apportioned to the license authorizations corresponding to clock category A blocks in that assignment phase market in proportion to the final clock phase price of each block. The assignment phase payment for the final assignment is apportioned to all blocks in that assignment phase market in proportion to the final clock phase price of each block.

Each license authorization calculation is rounded down to the nearest dollar and then the slack due to rounding down is distributed to authorizations (one dollar at a time) based on ascending order of final clock phase prices. Ties are broken based on ascending lexicographic order of license ID. License ID is defined as the PEA number followed by the letter and number representing the block in the final assignment (*e.g.*, PEA261-A1 or PEA001-C2).

9.3 Calculation of Net Per-License Prices

Net per-license prices are calculated by apportioning assignment phase payments and discounts to licenses in proportion to final clock phase prices.

Suppose that the bidder has been assigned a license in PEA j. Let k be the assignment phase market in which PEA j was assigned, that is, $j \in APM(k)$.

For a PEA j with a single assignment category, the net per-license price of a license in the PEA is determined by the following formula:

$$p_{T,i,\{ABC,j\}} + \frac{p_{T,i,\{ABC,j\}}}{\sum_{j' \in APM(k)} d_{T,i,\{ABC,j'\}} \cdot p_{T,i,\{ABC,j'\}}} \cdot (AP_{i,k} - D_{i,k})$$

That is, for each assignment phase market, its assignment payment and its discount (see Section 7.2 for how the discount is determined) are apportioned to the licenses in that assignment phase market in proportion to the final clock phase price of each license. Note that if the bidder does not qualify for a bidding credit and its assignment payment in an assignment phase market is zero, then the net per-license price of each license it is assigned in that market is simply the final clock phase price for that license.

For a PEA j with two assignment categories (interim and final), a winner is assigned a separate license authorization for each block won in clock category A (which includes an interim and a final assignment) and for each block won in clock category BC (which includes only a final assignment).

The net per-license price of a license authorization corresponding to a clock category A block won in PEA j is determined by the following formula:

$$p_{T,i,\{A,j\}} + \frac{p_{T,i,\{A,j\}}}{\sum_{j' \in APM(k)} d_{T,i,\{A,j'\}} \cdot p_{T,i,\{A,j'\}}} \cdot AP_{i,k,Interim} + \frac{p_{T,i,\{A,j\}}}{\sum_{j' \in APM(k)} (d_{T,i,\{A,j'\}} \cdot p_{T,i,\{A,j'\}} + d_{T,i,\{BC,j'\}} \cdot p_{T,i,\{BC,j'\}})} \cdot (AP_{i,k,Final} - D_{i,k})$$

The net per-license price of a license corresponding to a clock category BC block in PEA j is determined by the following formula:

$$p_{T,i,\{BC,j\}} + \frac{p_{T,i,\{BC,j\}}}{\sum_{j' \in APM(k)} (d_{T,i,\{A,j'\}} \cdot p_{T,i,\{A,j'\}} + d_{T,i,\{BC,j'\}} \cdot p_{T,i,\{BC,j'\}})} \cdot (AP_{i,k,Final} - D_{i,k})$$

That is, the assignment phase payment for the interim assignment is apportioned to the license authorizations corresponding to clock category A blocks won in that assignment phase market in proportion to the final clock phase price of each block. The assignment phase payment for the final assignment and the discount are apportioned to all license authorizations in that assignment phase market in proportion to the final clock phase price of each block.

Each license authorization calculation is rounded down to the nearest dollar, and then the slack due to rounding down is distributed to authorizations (one dollar at a time) based on ascending order of final clock phase prices. Ties are broken based on ascending lexicographic order of license ID.